**Chapter 5**

**Sequences and Series**

**5.3 The Divergence and Integral Tests**

**Section Exercises**

**For each of the following sequences, if the divergence test applies, either state that  does not exist or find  If the divergence test does not apply, state why.**

139. 

Answer:  Divergence test does not apply.

141. 

Answer:  Series diverges.

143. 

Answer:  (does not exist). Series diverges.

145. 

Answer:  Series diverges.

147. 

Answer:  does not exist. Series diverges.

149. 

Answer:  Series diverges.

151. 

Answer:  Divergence test does not apply.

**State whether the given  converges.**

153. 

Answer: Series converges, 

155. 

Answer: Series converges, 

157. 

Answer: Series converges,

**Use the integral test to determine whether the following sums converge.**

159. 

Answer: Series diverges by comparison with 

161. 

Answer: Series diverges by comparison with 

163. 

Answer: Series converges by comparison with 

**Express the following sums as  and determine whether each converges.**

165.  (*Hint:* )

Answer:  Since  diverges by 

167. 

Answer:  Since  diverges by 

**Use the estimate  to find a bound for the remainder  where **

169. 

Answer: 

171. 

Answer: 

**[T] Find the minimum value of  such that the remainder estimate  guarantees that  estimates  accurate to within the given error.**

173.  error 

Answer: 

175.  error 

Answer: 

177.  error

Answer: 

**In the following exercises, find a value of  such that is smaller than the desired error. Compute the corresponding sum  and compare it to the given estimate of the infinite series.**

179.  error  

Answer:  okay if  Estimate agrees with  to five decimal places.

181.  error  

Answer:  okay if   Estimate agrees with the sum to four decimal places.

183. Find the limit as  of  (*Hint:* Compare to )

Answer: 

**The next few exercises are intended to give a sense of applications in which partial sums of the harmonic series arise.**

185. In certain applications of probability, such as the so-called Watterson estimator for predicting mutation rates in population genetics, it is important to have an accurate estimate of the number. Recall that  is decreasing. Compute  to four decimal places. (*Hint:* )

Answer: 

187. [**T]** The simplest way to shuffle cards is to take the top card and insert it at a random place in the deck, called top random insertion, and then repeat. We will consider a deck to be randomly shuffled once enough top random insertions have been made that the card originally at the bottom has reached the top and then been randomly inserted. If the deck has  cards, then the probability that the insertion will be below the card initially at the bottom (call this card) is  Thus the expected number of top random insertions before  is no longer at the bottom is *n*. Once one card is below  there are two places below  and the probability that a randomly inserted card will fall below  is The expected number of top random insertions before this happens is  The two cards below are now in random order. Continuing this way, find a formula for the expected number of top random insertions needed to consider the deck to be randomly shuffled.

Answer: The expected number of random insertions to get  to the top is  Then one more insertion puts  back in at random. Thus, the expected number of shuffles to randomize the deck is 

189. Show that for the remainder estimate to apply on  it is sufficient that  be decreasing on , but  need not be decreasing on 

Answer: Set  and  such that  is decreasing on 

191. Does  converge ifis large enough? If so, for which 

Answer: The series converges for  by integral test using change of variable.

193. **[T]** A fast computer can sum one million terms per second of the divergent series  Use the integral test to approximate how many seconds it will take to add up enough terms for the partial sum to exceed 

Answer:  terms are needed.

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